INDIAN SCHOOL AL WADI AL KABIR

Pre-Board Examination 2

MATHEMATICS – 041-Set -1

QP. Code: 65/1/1

Roll No : | | | | | | | | |

Class : XII | Max Marks: 80
Date : 09.02.2025 | Time: 3 Hours

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question Paper divided into five Sections – A, B, C, D and E.
2. Section **A** has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.
3. Section **B** has 5 Very Short Answer (VSA)-type questions carrying 02 marks each.
4. Section **C** has 6 Short Answer (SA)-type questions carrying 03 marks each.
5. Section **D** has 4 Long Answer (LA)-type questions carrying 05 marks each.
6. Section **E** has 3 Case Based questions carrying 04 marks each.
7. There is no overall choice. However, an internal choice in 2 questions of 5 marks, 3 questions of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. This question paper contains 6 pages

SECTION – A (Each MCQ Carries 1 Mark)

1 The cofactor of (-1) in the matrix $\begin{bmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{bmatrix}$ is

a) -1 b) 0 c) 1 d) 2

2 The function f(x) = x^3 + 3x is increasing in ______

a) (– ∞, 0) b) (0, ∞) c) R d) (0, 1)

3 If X + Y = $\begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ and X - Y = $\begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$ then 2X – 3Y is

a) $\begin{bmatrix} 5 & 14 \\ 0 & -11 \end{bmatrix}$ b) $\begin{bmatrix} 5 & -14 \\ 0 & -11 \end{bmatrix}$ c) $\begin{bmatrix} -5 & -14 \\ 0 & -11 \end{bmatrix}$ d) $\begin{bmatrix} 5 & -14 \\ 0 & 11 \end{bmatrix}$

4 If y = $500e^{7x} + 600e^{-7x}$, then $\frac{d^2y}{dx^2}$ is

a) -7y b) 7y c) -49y d) 49y

5 $\int \frac{(1+\log x)^2}{x}$ dx = ______

a) $\frac{(1+\log x)^2}{2x} + c$ b) $\frac{(1+\log x)^3}{3x} + c$ c) $\frac{(1+\log x)^3}{3} + c$ d) $\frac{(1+\log x)^2}{3} + c$

6 $\sin^{-1}\left(\sin\frac{13\pi}{6}\right)$ = __________

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) π

7 The position vector of the point which divides the join of points with position vectors $2\vec{a} - 3\vec{b}$ and $\vec{a} + \vec{b}$ in the ratio 3 : 1 is

a) $\frac{3\vec{a} - 2\vec{b}}{2}$ b) $\frac{7\vec{a} - 8\vec{b}}{4}$ c) $\frac{3\vec{a}}{4}$ d) $\frac{5\vec{a}}{4}$

8 Solution of LPP
To maximize Z = 4x + 8y subject to constraints:
$2x + y \leq 30$, $x + 2y \leq 24$, $x \geq 3$, $y \leq 9$, $y \geq 0$ is

a) x = 12, y = 6 b) x = 6, y = 12 c) x = 9, y = 6 d) none of these

9 If m and n are the order and degree, respectively of the differential equation $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^4\right) = 0$, then the value of m + n is

a) 1 b) 2 c) 3 d) 4

10 If $\begin{bmatrix} 0 & -6 & 2 \\ a & c & -5 \\ b & 5 & 0 \end{bmatrix}$ is a skew symmetric matrix, then a+b+c:

a) 8 b) 4 c) -4 d) -10

11 If $y = \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)$, then $\frac{dy}{dx}$ is equal to

a) $\frac{1}{1 + x^2}$ b) $\frac{-1}{1 + x^2}$ c) $\frac{2}{1 + x^2}$ d) $\frac{-2}{1 + x^2}$

12 A set of values of decision variables that satisfies the linear constraints and non-negativity conditions of an L.P.P. is called its:

a) Unbounded solution
b) Optimum solution
c) Feasible solution
d) None of these

13 The angle between the lines 2x = 3y = –z and 6x = –y = – 4z is:

a) 0^0 b) 30^0 c) 45^0 d) 90^0

14 Let R be a relation on the set {1, 2, 3} given by R = {(1, 1), (2, 2), (1, 2), (2, 1), (2, 3)}. Which among the following element to be included to R so that R becomes Symmetric?

a) (3, 3) b) (3, 2) c) (1, 3) d) (3, 1)

15 A and B are invertible matrices of the same order such that $|(AB)^{-1}| = 8$, If $|A| = 2$, then $|B|$ is

a) 16 b) 6 c) $\frac{1}{6}$ d) $\frac{1}{16}$

16 The area enclosed by the circle $x^2 + y^2 = 8$ is

a) 16π sq units b) $2\sqrt{2}\pi$ sq units c) $8\pi^2$ sq units d) 8π sq units

17 If a line makes angles of 90°, 135° and 45° with the x, y and z axes respectively, then its direction cosines are:

a) $0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ b) $\frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$ c) $\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}$ d) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

18 If $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$, then the value of $|\vec{a} \times \vec{b}|$

a) 5 b) 10 c) 14 d) 16

Directions: In the following 2 questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false and R is True

19 **Assertion (A):** The matrix A $\begin{bmatrix} 1 & -1 & -2 \\ 1 & 0 & -3 \\ 2 & 3 & 0 \end{bmatrix}$ is a singular matrix.

Reason (R): For any square matrix A, $|A'| = |A|$

20 **Assertion (A):** Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$

Reason (R): Let E and F be two events with a random experiment, then $P(F/E) = \frac{P(E \cap F)}{p(E)}$

SECTION – B

(Each Question Carries 2 Marks)

21 Find the rate of change of volume of sphere with respect to its surface area, when radius is 2cm

22 (a) If $a = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right)$ and $b = \tan^{-1}(\sqrt{3}) - \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$, then find the value of a + b

- OR -

(b) Find the value of 'k' if $\sin^{-1}\left[k.\tan\left(2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right] = \frac{\pi}{3}$

23 Find the foot of the perpendicular drawn from the point (2, –1, 5) on the line

$\frac{x-11}{10} = \frac{y+2}{-4}\ \frac{z+8}{-11}$

24 (a) If vectors PQ = $-3\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$ and vector PR = $-5\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$ are the sides of a ΔPQR then find the angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$

- OR -

(b) If vectors PQ = $-3\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$ and vector PR = $-5\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$ are the sides of a ΔPQR then find the length of the median through the vertex P

25 If x sin(a + y) + sin a . cos(a + y) = 0 , then prove that $\frac{dy}{dx} = \frac{sin^2(a+y)}{\sin a}$

SECTION – C

(Each Question Carries 3 Marks)

26 Evaluate $\int_{-5}^{5} | x + 2 | \, dx$

- OR -

Evaluate $\int_{0}^{\frac{\pi}{2}} \frac{1}{1 + \sqrt{\tan x}} \, dx$

27 (a) A family has 2 children. Find the probability that both are boys, if it is known that

(i) at least one of the children is a boy.

(ii) the elder child is a boy.

- OR -

(b) Bag I contains 3 red and 4 black balls and bag II contains 4 red and 5 black balls. One ball is transferred from bag I to bag II and then a ball is drawn from bag II at random. The balls so drawn is found to be red in colour. Find the probability that the transferred ball is black.

28 Evaluate $\int \frac{1}{9x^2+6x+5}$ dx

29 Maximize Z = 600x + 400y Subject to the constraints:

$x + 2y \leq 12$; $2x + y \leq 12$; $4x + 5y \leq 20$; $x \geq 0$; $y \geq 0$ by graphical method

30 Evaluate $\int_{0}^{\frac{\pi}{2}} \sin 2x \, . tan^{-1}(\sin x) dx$

31 (a) Find the particular solution of the differential equation: $(1 + e^{2x})dy + (1 + y^2)e^x\, dx = 0$ given that y = 1 when x = 0.

- OR -

(b) Solve the following differential equation: $\frac{dy}{dx} + 2y.\tan x = \sin x$, given that y = 0, when $x = \frac{\pi}{3}$

SECTION – D

(Each Question Carries 5 Marks)

32 (a) The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

- OR -

(b) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$ then find A^{-1} and use it to solve the following system of the equations

$x + 2y - 3z = 6$, $3x + 2y - 2z = 3$, $2x - y + z = 2$

33 Make a rough sketch of the region given below and find its area using integration
$\{(x, y) : 0 \le y \le x^2 + 1,\ 0 \le y \le x+1,\ 0 \le x \le 2\}$

34 Let N denote the set of all natural numbers and R be the relation on N × N defined by (a, b) R (c, d) if ad (b + c) = bc (a + d). Show that R is an equivalence relation.

35 (a) Find the shortest distance between the lines
$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$.and $\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(\hat{i} - 3\hat{j} + 2\hat{k})$.

- OR -

(b) Answer the following:

(i) If $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$, then find the value of λ so that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular vectors.

(ii) Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $\widehat{3i} - 4\hat{j} + 4\hat{k}$ form the vertices of a right angled triangle.

SECTION – E

(CASE STUDY - Each Question Carries 4 Marks)

36 Read the following passage and answer the questions given below.
Tuba was doing a project related to the average number of hours spent on study by students selected at random. At the end of the survey, she prepared the report related to the data.
Let X denotes the average number of hours spent on study by students. The probability that X can take the values x, has the following form, where *k* is some unknown constant.

$$P(X = x) = \begin{cases} k, & if\ x = 0 \\ 2k, & if\ x = 1 \\ 3k, & if\ x = 2 \\ 0, & otherwise \end{cases}$$

Based on the above information, answer the following questions:

(i) What is the value of k? [1m]

(ii) What is the value P(X = 2)? [1m]

(iii) (a)What is the probability that average study time of students is at least 1 hour. [2m]

- OR -

(b) Find the mean of the given data. [2m]

37 The use of electric vehicles will curb air pollution in the long run. The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time ***t*** is given by the function $V(t) = \frac{t^3}{5} - \frac{5t^2}{2} + 25t - 2$ where ***t*** represents the time and t = 1, 2, 3.... corresponds to year 2001, 2002, 2003, ... respectively.

Based on the above information, answer the following questions:

(i) Can the above function be used to estimate number of vehicles in the year 2000. Justify [2m]

(ii) Prove that the function ***V(t)*** is an increasing function. [2m]

38 Bibek's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in figure. He has 200 ft of wire fencing.

Based on the above information, answer the following questions.

(i) If ***x*** denote the length of side of garden perpendicular to brick wall and ***y*** denote the length of side parallel to brick wall, then find the relation representing total length of fencing wire. [1m]

(ii) Write Area of the garden as a function of x, say A(x). [1m]

(iii) (a) For what value of ***x***, the value of A(x) is maximum. [2m]

- OR -

(b) Find the Maximum area of garden. [2m]

****

1	a) -1
2	c) R
3	a) $\begin{bmatrix} 5 & 14 \\ 0 & -11 \end{bmatrix}$
4	d) 49y
5	c) $\frac{(1+\log x)^3}{3} + c$
6	a) $\frac{\pi}{6}$
7	d) $\frac{5\vec{a}}{4}$
8	a) x = 12, y = 6
9	c) 3
10	c) -2, -7
11	c) $\frac{2}{1+x^2}$
12	c) Feasible solution
13	d) 90^0
14	b) (3, 2)
15	d) $\frac{1}{16}$
16	d) 8π sq units
17	a) 0, $\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$
18	d) 16
19	(c) A is true but R is false.
20	(a) Both A and R are true and R is the correct explanation of A.
21	$V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$ $\frac{dV}{dr} = 4\pi r^2$ and $\frac{dS}{dr} = 8\pi r$ $\frac{dV}{dS} = \frac{4\pi r^2}{8\pi r} = \frac{1}{2}r$ $\left.\frac{dV}{dS}\right\|_{r=2cm} = \frac{2}{2} = 1\ \text{cm}^3/1\text{cm}^2$

22

Ans: a = $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)+\cos^{-1}\left(-\frac{1}{2}\right)$

$=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+\cos^{-1}\left(-\frac{1}{2}\right)=\frac{\pi}{4}+\frac{2\pi}{3}=\frac{11\pi}{12}$

b = $\tan^{-1}(\sqrt{3})-\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)=\frac{\pi}{3}-\frac{2\pi}{3}=-$

Now a + b = $\frac{11\pi}{12}-\frac{\pi}{3}=\frac{11\pi-4\pi}{12}=\frac{7\pi}{12}$

- OR -

Ans: Given that $\sin^{-1}\left[k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]=\frac{\pi}{3}$

$\Rightarrow k\tan\left(2\cos^{-1}\left(\cos\frac{\pi}{6}\right)\right)=\sin\frac{\pi}{3}$

$\Rightarrow k\tan\frac{\pi}{3}=\frac{\sqrt{3}}{2}\Rightarrow k\sqrt{3}=\frac{\sqrt{3}}{2}\Rightarrow k=\frac{1}{2}$

23

$\cos\theta=\frac{\overline{PQ}.\overline{PR}}{|\overline{PQ}||\overline{PR}|}$

$\cos\theta=\frac{15+8+16}{\sqrt{9+16+16}\sqrt{25+4+16}}$

$\cos\theta=\frac{39}{\sqrt{41}\sqrt{45}}$

$\theta=\cos^{-1}\left(\frac{39}{3\sqrt{205}}\right)=\cos^{-1}\left(\frac{13}{\sqrt{205}}\right)$

- OR -

$\frac{\overline{PQ}+\overline{PR}}{2}=\frac{(-3i+4j+4k)+(-5i+2j+4k)}{2}$

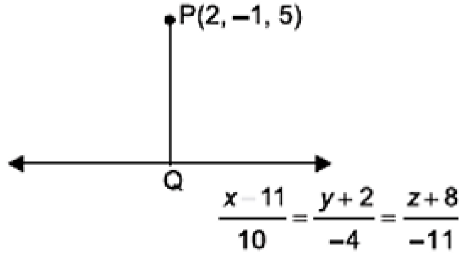
$\frac{-8i+6j+8k}{2}=4i+3j+4k$

The length of median = $|4i+3j+4k|=\sqrt{41}$

24

Ans. General point on the line $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}=\lambda$ is

$Q(10\lambda+11,\ -4\lambda-2,\ -11\lambda-8)$ ...(*i*)



Direction ratios of PQ are $10\lambda+11-2,\ -4\lambda-2+1,\ -11\lambda-8-5$

i.e. $10\lambda+9,\ -4\lambda-1,\ -11\lambda-13$

If PQ is perpendicular to the given line, then

$10(10\lambda+9)-4(-4\lambda-1)-11(-11\lambda-13)=0$

$\Rightarrow 237\lambda=-237\quad \lambda=-1$

Substituting in (i), we get the foot of perpendicular as Q(1, 2, 3).

Length of perpendicular PQ = $\sqrt{(2-1)^2+(-1-2)^2+(5-3)^2}=\sqrt{1+9+4}=\sqrt{14}$

25

$\Rightarrow x\sin(a+y)=-\sin a\cos(a+y)$

$\Rightarrow x=\frac{-\sin a\cos(a+y)}{\sin(a+y)}\Rightarrow x=-\sin a.\cot(a+y)$

Differentiating with respect to y, we get $\frac{dx}{dy}=-\sin a\left[-\operatorname{cosec}^2(a+y)\right].\frac{d}{dy}(a+y)$

$=-\sin a\left[-\operatorname{cosec}^2(a+y)\right].(0+1)=\frac{\sin a}{\sin^2(a+y)}$ $\qquad\Rightarrow\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$

- OR –

Let $I = \int_0^{\pi/2} \frac{dx}{1+\sqrt{\tan x}} = \int_0^{\pi/2} \frac{dx}{1+\sqrt{\frac{\sin x}{\cos x}}} = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ ...(i)

By the property, $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, we get

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}dx = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \quad ...(ii)$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2}\left[\frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} + \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\right]dx \quad = \int_0^{\pi/2} 1\cdot dx = [x]_0^{\pi/2} = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

26

$$|x+2| = \begin{cases} x+2, & x \geq -2 \\ -(x+2), & x < -2 \end{cases}$$

$$\int_{-5}^{5}|x+2|dx = \int_{-5}^{-2}-(x+2)dx + \int_{-2}^{5}(x+2)dx$$

$$= \quad -\left(\frac{x^2}{2}+2x\right)_{-5}^{-2} + \left(\frac{x^2}{2}+2x\right)_{-2}^{5}$$

$$-\left(\frac{4}{2}-4-\frac{25}{2}+10\right)+\left(\frac{25}{2}+10-\frac{4}{2}+4\right)$$

$$= \quad -\left(\frac{4-8-25+20}{2}\right)+\left(\frac{25+20-4+8}{2}\right) =$$

$$\frac{9}{2}+\frac{49}{2} = 29$$

27

$S = \{BB, BG, GB, GG\}$

(*i*) *A*: at least one of the children is a boy = *BB, BG, GB*

B: both are boys = *BB*

$A \cap B$: *BB*

both boys when at least one of the children is a boy

$$P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

(*ii*) *A*: the elder child is a boy = *BB, BG*

B: both are boys = *BB*

$A \cap B$: *BB*

Probability of the elder child is a boy.

$$P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}$$

- OR -

Ans: Bag I: 3 red + 4 black,
Bag II: 4 red + 5 black

Case I : when ball transferred is black.

$P(B/I) = \frac{4}{7}$

Total balls in bag II are 4 red + 6 black;

$P(R/II) = \frac{4}{10}$

Probability in this case $= \frac{4}{7} \times \frac{4}{10}$.

Case II: When ball transferred is red.

$P(R/I) = \frac{3}{7}$

Total balls in bag II are 5 red + 5 black,

$P(R/II) = \frac{5}{10}$

Probability in this case $= \frac{3}{7} \times \frac{5}{10}$

Using Bayes' Theorem, probability that the ball transferred is black

$$= \frac{\frac{4}{7} \times \frac{4}{10}}{\frac{4}{7} \times \frac{4}{10} + \frac{3}{7} \times \frac{5}{10}} = \frac{16}{16+15} = \frac{16}{31}$$

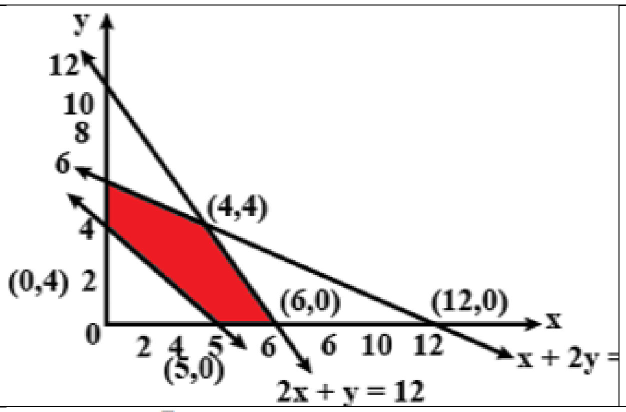
28

$$\int \frac{1}{9x^2+6x+5} dx = \frac{1}{9} \int \frac{1}{x^2+\frac{6}{9}x+\frac{5}{9}} dx$$

$$= \frac{1}{9} \int \frac{1}{x^2+\frac{2}{3}x+\frac{5}{9}+\left(\frac{1}{3}\right)^2-\left(\frac{1}{3}\right)^2} dx = \frac{1}{9} \int \frac{1}{\left(x+\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2} dx, \text{ putting } x+\frac{1}{3} = t \Rightarrow dx = dt$$

$$= \frac{1}{9} \int \frac{1}{t^2+\left(\frac{2}{3}\right)^2} dt = \frac{1}{9} \cdot \frac{1}{\frac{2}{3}} \tan^{-1}\left(\frac{t}{2/3}\right) + C = \frac{1}{6} \tan^{-1}\left[\frac{3\left(x+\frac{1}{3}\right)}{2}\right] + C = \frac{1}{6} \tan^{-1}\left(\frac{3x+1}{2}\right) + C$$

29



Corner points	Z = 600x + 400y
(0,4)	1600 minimum
(0.6)	2400
(4,4)	4000 maximum
(6,0)	3600
(5,0)	3000

30

Let $I = \int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)\, dx$

$= \int_0^{\frac{\pi}{2}} 2 \sin x \cos x \tan^{-1}(\sin x)\, dx$

let $\sin x = t \quad \Rightarrow \cos x\, dx = dt$

When $x = 0, t = 0$, when $x = \frac{\pi}{2}, t = 1$

$\Rightarrow I = 2 \int_0^1 t \tan^{-1}(t)\, dt \quad$ ...(i)

$\int t \cdot \tan^{-1} t\, dt =$

$= \tan^{-1} t \cdot \int t\, dt - \int \left\{\frac{d}{dt}(\tan^{-1} t) \int t\, dt\right\} dt$

$= \tan^{-1} t \cdot \frac{t^2}{2} - \int \frac{1}{1+t^2} \cdot \frac{t^2}{2} dt$

$= \frac{t^2 \tan^{-1} t}{2} - \frac{1}{2} \int \frac{t^2+1-1}{1+t^2} dt$

$= \frac{t^2 \tan^{-1} t}{2} - \frac{1}{2} \int 1\, dt + \frac{1}{2} \int \frac{1}{1+t^2} dt$

$= \frac{t^2 \tan^{-1} t}{2} - \frac{1}{2} \cdot t + \frac{1}{2} \tan^{-1} t$

$\Rightarrow \int_0^1 t \cdot \tan^{-1} t\, dt = \left[\frac{t^2 \cdot \tan^{-1} t}{2} - \frac{t}{2} + \frac{1}{2} \tan^{-1} t\right]_0^1$

$= \frac{1}{2}\left[\frac{\pi}{4} - 1 + \frac{\pi}{4}\right] = \frac{1}{2}\left[\frac{\pi}{2} - 1\right] = \frac{\pi}{4} - \frac{1}{2}$

$I = 2\left[\frac{\pi}{4} - \frac{1}{2}\right] = \frac{\pi}{2} - 1$

31 We have, $(1 + e^{2x})\, dy + (1 + y^2)\, e^x\, dx = 0$ and given that $y = 1$, when $x = 0$

$$\therefore \quad \frac{dy}{dx} = \frac{-(1+y^2)e^x}{1+e^{2x}} \qquad \Rightarrow \frac{dy}{-(1+y^2)} = \frac{e^x dx}{1+e^{2x}}$$

Integrating both sides, we get

$$-\int \frac{dy}{1+y^2} = \int \frac{e^x dx}{1+e^{2x}} \qquad \Rightarrow -\tan^{-1} y = \int \frac{e^x dx}{1+(e^x)^2}$$

$$\Rightarrow \; -\tan^{-1} y = \int \frac{dt}{1+t^2} \qquad [\text{Putting } e^x = t \;\Rightarrow\; e^x dx = dt]$$

$$\Rightarrow -\tan^{-1} y = \tan^{-1}(t) + C \quad \Rightarrow \; -\tan^{-1} y = \tan^{-1}(e^x) + C$$

Put $x = 0, y = 1$ in (i), we get

$$-\tan^{-1} 1 = \tan^{-1}(e^0) + C \qquad \Rightarrow -\frac{\pi}{4} = \frac{\pi}{4} + C \;\Rightarrow\; C = -\frac{\pi}{2}$$

Putting the value of C in (i), we get

$$-\tan^{-1} y = \tan^{-1}(e^x) - \frac{\pi}{2} \qquad \Rightarrow \frac{\pi}{2} = \tan^{-1}(e^x) + \tan^{-1} y$$

Hence, $\tan^{-1}(e^x) + \tan^{-1} y = \frac{\pi}{2}$ is the required solution.

- OR -

Comparing it with $\frac{dy}{dx} + Py = Q$, we get $P = 2\tan x, Q = \sin x$

$$\therefore \quad \text{IF} = e^{\int 2\tan x dx} = e^{2\log\sec x} = e^{\log\sec^2 x} = \sec^2 x \qquad [\because e^{\log z} = z]$$

Hence, general solution is $y.\sec^2 x = \int \sin x.\sec^2 x dx + C$.

$$y.\sec^2 x = \int \sec x.\tan x\, dx + C \;\Rightarrow\; y.\sec^2 x = \sec x + C \;\Rightarrow\; y = \cos x + C\cos^2 x$$

Putting $y = 0$ and $x = \frac{\pi}{3}$, we get $0 = \cos\frac{\pi}{3} + C.\cos^2\frac{\pi}{3}$

$$\Rightarrow 0 = \frac{1}{2} + \frac{C}{4} \quad \Rightarrow \quad C = -2$$

$\therefore$ Required solution is $y = \cos x - 2\cos^2 x$.

32 $x + y + z = 6$

$y + 3z = 11$

$x + z = 2y$ or $x - 2y + z = 0$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \;\&\; B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$|A| = 1(1 + 6) - 0 + 1(3 - 1) = 9$

$$\text{Hence, } adj(A) = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$\text{Thus, } A^{-1} = \frac{1}{|A|} adj(A) = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$\therefore X = A^{-1}B$

$$\Rightarrow X = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 9 \\ 18 \\ 27 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow x = 1, y = 2, z = 3$$

- OR -

$$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{vmatrix}$$

$= 1(2-2) - 2(3+4) - 3(-3-4)$

$= -14 + 21 = 7$

$$\therefore \ \text{adj } A = \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$$

The given system of equations is $x + 2y - 3z = 6$

$3x + 2y - 2z = 3$

$2x - y + z = 2$

The system of equations can be written as $AX = B$

$$\text{where } A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

$\because$ A^{-1} exists, so system of equations has a unique solution given by $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 7 \\ -35 \\ -35 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}$$

$\Rightarrow x = 1, y = -5, z = -5$

33 Ans: Given: $y^2 = 2x$

$y = x - 4$

Required area is OABCO

from (1) and (2), $(x-4)^2 = 2x$

$\Rightarrow x^2 - 10x + 16 = 0$

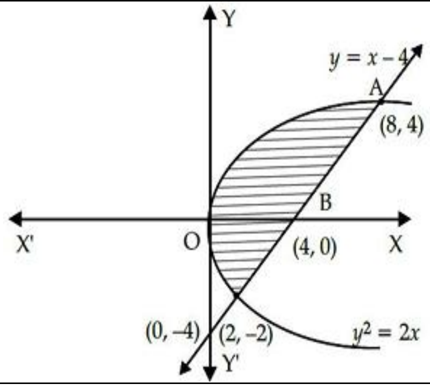
$\Rightarrow (x-8)(x-2) = 0$

$\Rightarrow x = 8$ and $x = 2$

$\therefore$ Intersection points $(2, -2)$ and $(8, 4)$

$$\text{Area} = \left[\frac{y^2}{2} + 4y - \frac{y^3}{6}\right]_{-2}^{4} = \left(8 + 16 - \frac{32}{3} - 2 + 8 - \frac{4}{3}\right) = 30 - 12$$

$= 18 \text{ unit}^2$



34 Ans: Relation R on $N \times N$ is given by

$(a, b)\ R(c, d) \Leftrightarrow ad(b + c) = bc(a + d)$

For reflexive:

For $(a, b) \in N \times N$

$(a, b)\ R(a, b) \Rightarrow ab(b + a) = ba(a + b)$

true in N

Hence, reflexive

For symmetric:

For $(a, b), (c, d) \in N \times N$

$(a, b)\ R(c, d) \Rightarrow ad(b + c) = bc(a + d)$

$\Rightarrow cb(d + a) = da(c + b)$ (×

$\Rightarrow (c, d)\ R(a, b)\ \forall\ (a, b), (c, d) \in N \times N.$

Hence, symmetric

For transitive:

For $(a, b), (c, d), (e, f) \in N \times N$

Let $(a, b)\ R(c, d)$ and $(c, d)\ R(e, f)$

$\Rightarrow ad(b + c) = bc(a + d)$

$$\Rightarrow \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$$

and $cf(d + e) = de(c + f)$

$$\frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$$

$$\Rightarrow \frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$$

$$\Rightarrow \frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f}$$

$af(e + b) = be(f + a)$

$\Rightarrow af(b + e) = be(a + f)$

$\Rightarrow (a, b)\ R(e, f)$

As $(a, b)\ R(c, d), (c, d)\ R(e, f)$

$\Rightarrow (a, b)\ R(e, f)$ Hence, transitive.

As relation R is reflexive, symmetric and transitive. Hence, R is an equivalence relation.

35 i) $\overline{a_1} = i + 2j + 3k$; $\overline{b} = i - 3j + 2k$

$\overline{a_2} = 4i + 5j + 6k$ Both lines are parallel.

$$\bar{a}_2 - \bar{a}_1 = 3i + 3j + 3k$$

$$\bar{b} \times (\bar{a}_2 - \bar{a}_1) = \begin{vmatrix} i & j & k \\ 1 & -3 & 2 \\ 3 & 3 & 3 \end{vmatrix}$$

$$= i(-9-6) - j(3-6) + k(3-(-9))$$

$$= -15i + 3j + 12k$$

Shortest distance = $\left| \dfrac{|\bar{b} \times (\bar{a}_1 - \bar{a}_2)|}{|\bar{b}|} \right|$

$$= \left| \frac{|-15i + 3j + 12k|}{|i - 3j + 2k|} \right|$$

$$= \left| \frac{\sqrt{225 + 9 + 144}}{\sqrt{1 + 9 + 4}} \right| = \frac{\sqrt{378}}{\sqrt{14}}$$

$$= \frac{3\sqrt{3}\sqrt{14}}{\sqrt{14}} = 3\sqrt{3}$$

OR

(i)

Ans: Given that $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$

$\therefore\ \vec{a} + \vec{b} = \hat{i} - \hat{j} + 7\hat{k} + 5\hat{i} - \hat{j} + \lambda\hat{k} = 6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}$

and $\vec{a} - \vec{b} = \hat{i} - \hat{j} + 7\hat{k} - 5\hat{i} + \hat{j} - \lambda\hat{k} = -4\hat{i} + (7-\lambda)\hat{k}$

Now, $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular vectors

$\Rightarrow (\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 0$

$\Rightarrow (6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}).(-4\hat{i} + (7-\lambda)\hat{k} = 0$

$\Rightarrow -24 + 0 + (7+\lambda)(7-\lambda) = 0$

$\Rightarrow -24 + 49 - \lambda^2 = 0 \Rightarrow \lambda^2 = 25 \Rightarrow \lambda = \pm 5$

(ii)

$\overrightarrow{AB} = (\hat{i} - 3\hat{j} - 5\hat{k}) - (2\hat{i} - \hat{j} + \hat{k}) = -\hat{i} - 2\hat{j} - 6\hat{k}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{1 + 4 + 36} = \sqrt{41}$

$\overrightarrow{BC} = (3\hat{i} - 4\hat{j} - 4\hat{k}) - (\hat{i} - 3\hat{j} - 5\hat{k}) = 2\hat{i} - \hat{j} + \hat{k}$

$\Rightarrow |\overrightarrow{BC}| = \sqrt{4 + 1 + 1} = \sqrt{6}$

and $\overrightarrow{AC} = (3\hat{i} - 4\hat{j} - 4\hat{k}) - (2\hat{i} - \hat{j} + \hat{k}) = \hat{i} - 3\hat{j} - 5$

$\Rightarrow |\overrightarrow{AC}| = \sqrt{1 + 9 + 25} = \sqrt{35}$

$\therefore |\overrightarrow{AB}|^2 = |\overrightarrow{AC}|^2 + |\overrightarrow{BC}|^2$

Hence, ABC is a right angled triangle.

36 Ans: (i) Since, $\sum P(x) = 1$

$\therefore k + 2k + 3k + 0 = 1$

$\Rightarrow 6k = 1 \Rightarrow k = 1/6$

(ii) From (i), $k = 1/6$

$\therefore P(x = 2) = 3k = 3/6 = 1/2$

(iii) If a students has study time at least 1 hr then either he/she has studies for 1 hour or 2 hour.

$\therefore$ Required Probability = $P(x = 1) + P(x = 2) = 2k + 3k = 5k = \frac{5}{6}$

Mean = $\sum xP(x) = 0 \times \frac{1}{6} + 1 \times \frac{2}{6} + 2 \times \frac{3}{6} = 0 + \frac{2}{6} + 1 = \frac{1}{3} + 1 = \frac{4}{3}$

37 Ans: (a) $V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$

$\because$ For 2000, t = 0

$\therefore V(0) = 0 - 0 + 0 - 2$

Since, number of vehicles cannot be negative.

Therefore, the given function cannot be used to estimate number of vehicles in the year 2000.

(b) $V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$

$$V'(t) = \frac{3}{5}t^2 - 5t + 25 = \frac{3}{5}\left[t^2 - \frac{25}{3}t + \frac{125}{3}\right]$$

$$= \frac{3}{5}\left[\left(t - \frac{25}{6}\right)^2 - \frac{625}{36} + \frac{125}{3}\right] = \frac{3}{5}\left[\left(t - \frac{25}{6}\right)^2 + \frac{875}{36}\right]$$

$V'(t) > 0$ for any value of t.

$\therefore$ The given function $V(t)$ is an increasing function.

38 (i)

$\therefore$ x + y + y + perimeter of semicircle = 10

$\Rightarrow x + 2y + \pi\frac{x}{2} = 10$ which is the relation between x and y

	(ii)	Area $A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 = x\left(5 - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{1}{2}\frac{\pi x^2}{4}$ $= 5x - \frac{x^2}{2} - \frac{\pi x^2}{4} + \frac{\pi x^2}{8} = 5x - \frac{x^2}{2} - \frac{\pi x^2}{8}$
	(iii)-a	We have, $A = 5x - \frac{x^2}{2} - \frac{\pi x^2}{8}$ $\Rightarrow \frac{dA}{dx} = 5 - x - \frac{\pi x}{4}$ $\frac{dA}{dx} = 0 \Rightarrow 5 - x - \frac{\pi x}{4} = 0$ $\Rightarrow 5 = x + \frac{\pi x}{4} \Rightarrow x(4+\pi) = 20 \Rightarrow x = \frac{20}{4+\pi}$
	(iii)-b	(iii) We have, $y = 5 - \frac{x}{2} - \frac{\pi x}{4} = 5 - x\left(\frac{1}{2} + \frac{\pi}{4}\right)$ $= 5 - x\left(\frac{2+\pi}{4}\right) = 5 - \left(\frac{20}{4+\pi}\right)\left(\frac{2+\pi}{4}\right) = 5 - 5\left(\frac{2+\pi}{4+\pi}\right) = \frac{20 + 5\pi - 10 - 5}{4+\pi}$